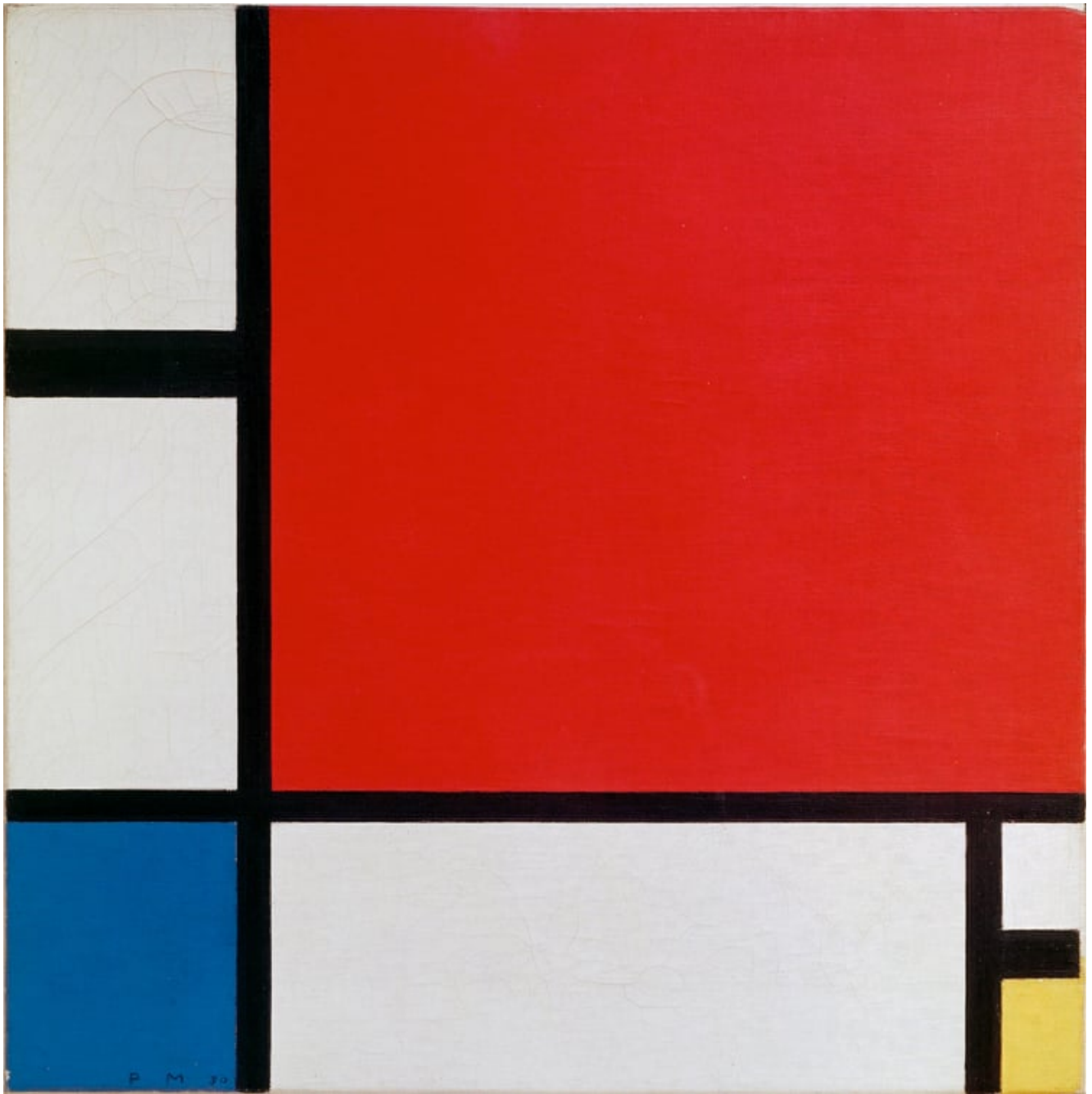# The Secret of Mathematics

Composition with Red, Blue, and Yellow, Piet Mondrian, 1930

Mathematics is a human accomplishment which has been around for more than twenty-five centuries, though it has been slipping largely unnoticed into a state of public invisibility during the last sixty years. It used to be generally regarded as the Heartland of Truth, because it is timeless, and its conclusions can be checked, 2, 20 or 200 times—to make absolutely sure they are correct. In spite of this copper-

bottomed status, though, it was conspicuously absent from the fierce debate about the dangers of post-truth public policy, which came to the fore after Donald Trump entered the White House. No one, it appeared, was willing to point out that mathematics has discovered many millions of important, absolute truths during its long sojourn (> 2,000 years) as an admired subject. Why did no one make this point? Probably because they were aware that this former admiration was fading away.

So what is mathematics all about? What is the meaning of mathematics? Why has it been sliding towards invisibility during the last six decades?

The answer to the first two questions is, that those who are supposed to be its most perceptive leaders, don't know.

The answer to the third question is that these leaders rolled over in the 1960s, and let the computer industry steal all the credit for mathematics' public work. (The most spectacular work was bringing astronauts back from the Moon, using no more fuel than a full tank in a family car.)

Since the start of the computer age—the automation of mathematics age—all serious real-world mathematical problem-solving has been handled, naturally, on computers. These machines have implemented the mathematics needed to find the solutions. And thanks to the capitulation of the mathematic leadership, it is the *machines* which have been credited with the achievement—not the *brains* who worked-out which equations were needed to find the answers. This is like treating a *bicycle* as being the real winner of the *Tour de France*, not the athlete who made the effort. The mathematic establishment could easily have disputed this computer steal, but they did nothing.

The establishment has also managed to throw away maths' former status as the Heartland of Truth. They introduced fog into the

house of mathematics in the 1920s when they decided to contradict their own subject and decree that a set could never be a member of itself. They were saying, in effect, that the set of all sets mentioned in this article is not a set mentioned in this article. It was denial of the most obviously *obvious* logical commonsense.

So … what is mathematics all about? Well, a notable American mathematician, Reuben Hersh, wrote a large, thoughtful, much-praised book *What is Mathematics, Really?* in 1996. He commented in the Preface that he was repudiating the orthodox platonism and formalism favoured by the subject's conservative leaders. But all he could offer in their place was that mathematics was "a human activity … part of human culture … intelligible only in a social context." I'm afraid this does not tell us very much.

Actually the real source of the meaning of mathematics was pin-pointed—though not well publicised—by the great American philosopher Charles Peirce, probably originally in the 1890s. He pronounced mathematics to be the "Science of hypothesis." But he couldn't get his insight noticed.

Peirce was a sad victim of moralistic Victorian attitudes in 19th century New England. As a young man, growing up on the campus of Harvard—where his father was a leading figure—he became involved in an amorous ex-marital affair and was subsequently banned for life from polite company. His collected papers were only published in the mid 1930s, and his brilliant insight into the nature of mathematics only became (weakly) known after the publication of one of his essays in *The World of Mathematics* (1956). Thereafter, however, it reverted to obscurity and was rarely picked up, probably because the mathematic establishment did everything they could to hush it.

Peirce's definition of mathematics applies to both pure and applied mathematics. In the pure case it amounts to saying

that the best advances in pure maths arise mainly from researchers trying to prove famous conjectures like those of Fermat and Riemann. These are hypotheses to the effect that, what appears to be true in all the known cases, is indeed a universal law.

But it is in applicable mathematics that Peirce's insight really bites. By saying that mathematics is "the science of hypothesis," he is saying that the application of mathematics to tease-out the implications of putative theories in science and in public projects is its *main raison d'etre*. This is where mathematics generates priceless *vision*, or if you prefer, a rounded, in-depth picture of what the hypothesis implies (in science) or how it would work out (in innovative projects).

This is where the service provided by mathematics makes its greatest, and most obvious, contribution to human well-being. In the Graeco-Roman world, mathematics was used in this mode mainly to plan military campaigns, engineering feats such as aqueducts, and designing triumphant monuments. This is what kept the Roman Empire on the road. (The "maths" used was mostly a bundle of what we would call "logistics.") Today we have a phrase for this side of mathematics: *mathematical modelling*.

There is no doubt that military edge is the principal reason why maths was held in high esteem by Emperors, Tyrants, Generals, etc. for twenty-five centuries. It was the reason they gave the top mathematicians social status. After the 17th century, the practice of mathematical modelling began to involve more advanced mathematics such as calculus: and the machines it enabled began to change the world.

So why did the eminent, highly intelligent gurus of professional mathematics manage to overlook this simple truth for two and a half millennia? They were the people most involved in the logistic modelling needed to plan projects

like the pyramids, the Corinth canal, the tomb of Mausolos, the lighthouse of Alexandria … They were the ones who benefitted most from mathematics' high social standing.

Well, we have to face the home truth that the mathematicians were single-minded virtuoso performers, not seers. The practice of mathematical modelling also involves realistic imagination, something which these virtuosi didn't practise, and for which they were probably least equipped. (They were often clever, reality-phobic individuals, who preferred to work in the narrow, stable, ordered, predictable world of numbers, rather than in the wide-open, unstable, risky, ambiguous world of ordinary life.)

It is ironic that today the Pandemic has finally allowed the public frequently to hear the term 'mathematical model,' which was quietly sidelined for sixty years by the mathematics establishment.

Mathematical modelling should be at the centre of the maths taught in school, because it widens and energises the mind. It fields, and arises from, vision and imagination—which is the main source of the subject's importance, and the real reason for having it in the curriculum. We are reminded of a mantra of the early 1980s when the disaster of "New Maths for Schools" had finally become plain: "Mathematics is too important to be left to the mathematicians!"