The Two-Sidedness of Mathematics

Monk by the Sea by Caspar David Friedrich, 1808/1810

by Christopher Ormell (March 2022)

Mathematics, regarded as a force for good, has two quite separate –much misunderstood, but potentially inspiring– sides, each with its own agenda: (1) to be the Heartland of Truth, and (2) to be the Pathfinder for Progress. (The 'Heartland of Truth' refers of course to "indicative" or "objective" truth, not to emotive truths.) I explored its meaning in my article in the January issue. 'Pathfinder for Progress' is a succinct description of the role of mathematics when it is interpreted via the insight of Charles Peirce, as outlined in my article in the November issue. Peirce saw that mathematics was <<the science of hypothesis>> and progress in any shape or form can only begin as an hypothesis. When an hypothesis is capable of being rendered into a mathematical form, the maths enables us to tease out its hidden

(predictable) implications. This reveals details lost in the mazy complexity of the original idea.

Unfortunately, neither side of maths is in good shape at the present time. The mathematical "priesthood" –which still determines the Official Line about mathematics– seems to harbour a slight commitment to mystification… one which is preventing its members from seeing that the Heartland has gone to seed. And, of course, they long ago washed their hands of any involvement with the problems of real life, let alone progress. Both sides of maths also seem to have been snuffed out by an all-pervading public mental fog associated with post-modern disillusion.

These two sides of mathematics, the intentional (Truth) and the extensional (Progress) are both important. They are likely to be specially favoured respectively by conservatives and reformers. But in the broad sense that the subject-as-a-whole has *meaning,* this evidently springs mainly from the progress side, because this deals with promising aspects of the real world, which of course, carry future down-to-earth hopes. Hypothesis exploration can stand on its own feet, whereas the 'The Truth' side has latterly been allowed to become overgrown, abstruse and, arguably, somewhat artificial. It now looks too much like a scholastic in-house project to be the 'Heartland' of anything.

So, it is arguably an awareness of the extraordinary modelling power of mathematics which effectively legitimises and underpins the validity of the accessible, lucid, pure mathematics which is capable of being the 'Heartland of Truth'. ('Heartland' because its indicative truths are the best, most precise, most well-defined and most unexpected, we can ever meet.)

This is not part of the rhetoric favoured by today's gurus, though. They have done their damndest to try to portray the motivation for modelling as narrow, materialistic, and self-

servingly commercial. To counter this slur, it needs to be pointed out that modelling mathematics has made great contributions in the past to the *common good* (e.g., navigation at sea, the laws of physics, the invention of the computer, the unravelling of DNA…). It still pursues this role today in full measure, when it is used by NASA, Covid epidemiologists, weather forecasters, etc.

So, the malicious ploy that modelling is essentially self-serving and "materialistic" needs to be nailed. It amounts to no more than black propaganda, put out by dyed-in-the wool gurus, who have never bothered even to try to understand the challenges and cognitive rewards of mathematical modelling.

The upshot is that mathematical modelling has been absent from the Official Line during recent times. This has created some confusion about what 'mathematical modelling' really means. It has no connection whatever with fashion, or photo, modelling: it isn't a kind of showing-off, or even essentially a "method of communication" (though it can be communicated). A mathematical model begins as an outline, skeletal representation of a projected project, phenomenon, process or desired state of affairs. ('Skeletal' because much surface detail can be ignored.) It then uses mathematic operations to explore this hypothetical state of affairs –usually a proposed practical project or a theory– to see what the outcome is likely to be. In effect the "model" is simulating (mimicking) the essentials of the proposed real situation –across many varied manifestations. Working the model enables us to see, by searching, what the main significant implications would be *if* the project were carried out, or if the theory were adopted as valid science. Using it is like poring over a map of a hilly area to find the best way to hike from A to B.

Progress happens when far-sighted people have inspiring visions of the future. They see in their imagination that things *might* be improved in a specific way. But imagination is only an imprecise, outline faculty at the best of times. It

tends to overlook a lot of detail, which of course is potentially “small print”, capable of harbouring problems. This why it needs to be followed up with a rigorous mathematical embodiment of the envisaged project, which can then be driven to simulate the idea’s predictable possibilities. (Modelling can only “bring out” the hidden *predictable* core aspects of the original idea.) Doing it is thus a “feasibility study”: mathematic operations on the model mimicking what would be the result of doing things in the proposed situation. Imagination sets the problem, but it is not capable of focusing onto the nitty-gritty. It serves like a low powered telescope used by astronomers to align a high-powered telescope.

On the Heartland side, maths can only live up to this billing if a healthy sub-body of the educated public are actually participating with genuine satisfaction in the cognitive release (i.e., the *Eureka feeling*) it is capable of providing. Sadly, it is not being organised today to have this thrilling effect. In schools, pure mathematics often nowadays tends to reduce to a resented ritual, a chore, a nod towards past attitudes… which have long since expired. The current PR of the maths establishment hypes apparent, but intrinsically obscure, “triumphs” some gurus have, apparently, recently pulled off in the logical stratosphere.

The reality on the ground, though, is that the very notion of pure maths as society’s Heartland of Truth, has faded badly in recent years. It was always dependent on pure mathematics being seen widely as a valued logos. When this was still around, it naturally served as the Heartland of Truth, because its terms were all thoroughly rational, lucid and rigorously defined. In a word, the most luminous indicative truths are to be found in mathematics. This gives the truths of accessible maths an edge which is not present in the case of empirical truths. For example, <<water flows downhill>> represents the residue of repeated experience of potentially

complex, ill-defined circumstances. (How do you begin to *define* 'downhill', 'flows' and 'water'?)

Nor is the concept of maths as our chief Pathfinder for Progress faring any better. It is a way of regarding mathematics as a search for clear vision in obscure, unfamiliar, practical and theoretical situations. But maths first has to be seen by the average intellectually active person as operable in this way. The gurus of the maths hierarchy, though, have notoriously portrayed this "applicable maths" as <<low status, far below their superior level of reasoning>>. Their clout in the media has then ensured that many people have turned off. This underwhelming "Official" verdict on the subject's modelling side, makes a big difference to how it is seen. Using maths to do exciting exploratory modelling was the source of the subject's former applicative thunder for teenagers. Foolishly, it was given away by the maths hierarchy to the computer industry in the 1960s. (They let the computer industry claim that the magic originated from their machines.) In schools this give-away has removed the ground from under the feet of conscientious teachers who are minded to convey the full meaning of maths to their students. For more than sixty years the Official Story has been that it is <<*computers* which do the applications>>, a gross misreading of the situation, no more credible than the claim that it is the *bicycle* that wins the Tour de France, or that the ascent of Everest was the result of using of *ladders*. Computers don't know which equations need to be solved, which unknowns need to be found, in which order. Nor do they know the significance of the results.

It doesn't have to be like this. A first step –in re-interpreting maths– will be to establish firm rigour protocols for any 'mathematical modelling' presented in public. There is a home truth here: mathematical modelling only works well when it is based on thoroughly reliable, trusted empirical principles. It can give us unexpected predictions, but only

if the empirical principles built into the model at the beginning are thoroughly sound.

So professional modelling with mathematics has to reflect –-if it is to be trusted and valued–- the highest standards of empirical rigour. But because modelling's thunder was given away –and also effectively kept out of the ordinary public's sight for sixty years– any general discussion of maths' meaning tends to start on the wrong foot. Where the phrase 'mathematical modelling' does still carry a little –often misunderstood– clout –-e.g., in the Financial Sector and Marketing–- there have been too many cowboy operators who have blandly neglected to do a quarter of the checking of the empirical assumptions. These lapses have produced much disillusion, and even given mathematical modelling a bad name in some quarters.

So, the gurus of mathematics have, by sticking to their outmoded attitudes, unwittingly distorted their subject's point and meaning. This can be seen clearly today, because we are now in a post-Belief era, in which the spiritually motivating effect of maths' alleged former deep association with religion has disappeared. This has left the mathematical 'guru priesthood' out on a limb. One might have expected that a new *take* on maths would begin to emerge, and especially after the deeply embarrassing fiasco –New Maths for Schools in the 1960s– which was a folly engineered by the gurus. But there are still quite a few influential people in academia, the media, judiciary, politics, etc. who –-even today–- are still treating the leading gurus as *the greyest* of the grey eminences.

This lingering notion of the gurus as seers, is not justified by the facts. It probably occurs because the gurus have long since skewed maths education in a way which *puts off* a majority of mentally lively people during their formative school years… leaving them in a limbo. And when these people later become influential voices in academia, politics, etc,

they don’t feel they have any kind of basis for criticising the gurus, about what they have been brainwashed into thinking must be the hardest subject of all. Therefore, their default disposition remains… tacitly to accept what the gurus say.

Because abstraction is a strain, the gurus have enjoyed almost total control of the way the subject is taught in schools since ancient times. They have moulded it inevitably towards their own preconceptions. This has made it peculiarly unsuitable for the average intelligent person. The Pythagoreans started off in the sixth century BCE believing that <<God had made a mistake when He created the universe!>> (sic), and it took them a long time to rid themselves of this silly conceit. In the modern era they have similarly stuck to other defiant notions which include the assumption that “not finite” can occur with degrees of “notness”… and that their subject is “culturally far superior” to others. Such hubris allows them to brush aside any feedback they might receive from the rest of lay society. The gurus are mostly not aware that their subject has become the black hole in the curriculum in many schools. They have foolishly allowed ordinary school mathematics to wither in the wind. A way *is* emerging to turn over a new leaf, but it will take much effort to complete the job.

POSTSCRIPT

All this might be interpreted as conveying the message that <<mathematics is hunky-dory, after all!>>. But it is not the whole story. Mathematics harbours a serious *downside* when interpreted as *the* Pathfinder for Progress –it is only capable of simulating fully determined, fully predictable, change. As a result, too much mathematics in one’s mental diet can give rise to a dangerous false impression: that everything in the future is to be determined, predictable and relentless. This has the oppressive effect that it is saying that rigid rules

and regimentation are always the inevitable, or even the desirable, end. We know just how evil the consequences which flow from this can be. In the Ancient World the silver mines of Roman Spain were death-traps, and the Slavery of the 18th century was another example of inexcusably maths-mesmerised, brutal, civic power. In my next essay I reveal that Peirce's insight about mathematics has a surprising, wholly unexpected, upshot –that there is room for a new, contra-mathematic, 100% abstract, 100% lucid, logos, which will be the science of those possibilities (hypotheses) which are inherently flexible, diverse and which naturally field random variability. Mathematics then ceases to be *the* Pathfinder for Progress and becomes just one of two rival abstract languages which can be used to explore the potential shape of the future.

TABLE OF CONTENTS

Christopher Ormell is an older philosopher of mathematics. He studied mathematics and philosophy at Magdalen College, Oxford, and has been campaigning for years for the Peircean interpretation of maths to be recognised as its main source of meaning.

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